Evaluating predator prey models

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Introduction

This document provides a guide for your project. It is not citeable material and is written in an informal style to provide advice on how to address the project. Do not quote directly from it, but do feel free to rewrite the ideas expressed in your own words following the style of a scientific paper.

Introduction

The response to the pandemic of 2020/21 was heavily influenced by the use of ecological modelling in order to predict the dynamics of a complex system. Much of the modelling was derived from work conducted by a team of modellers at Imperial college(Ferguson et al. 2006). This has heightened public awareness of both the strengths and potential weaknesses of ecological modelling. A particular challenge arises when models are used to predict the result of some change to a complex system in the future. As data on the future result of the intervention are clearly not available, the model cannot be validated directly. In many respects expecting any model to predict the future is asking far too much. Ecological models provide frameworks for thought. If used carefully they can be useful for improving understanding of a complex system. If used without understanding they may produce misleading results. This directed IRP provides an opportunity to look at ecological modelling in some depth. There is a very rich literature on the subject to be explored with many differing viewpoints and many critiques of models.

The IRP is based on the use of an adapted classical ecological model to predict the impact of introducing pine martens in order to control grey squirrels. Modelling provides a good source of material for a project as, unlike field work, models can be relied upon to produce "data." The thought that goes into parameterising a model can be at least as useful as the model results themselves, and often much more so. All models are wrong, but some are useful. They are only useful if their limitations are well understood.

The Lotka Volterra predator prey model

The classic "text book" model that links the dynamics of a prey population to that of a predator population was developed by Lotka and Volterra in 1927 (Berryman, Alan A 1992). It is extremely simple.

$\frac{dN}{dt} = r_1 N - \alpha NP$

$\frac{dP}{dt} = \alpha\beta NP - \gamma P$

There are many explanations of the derivation of these equations in text books and on the internet. They can be written in several different ways, but the basic idea is always the same. I'll explain it in simple terms here.

$\frac{dN}{dt}$ and $\frac{dP}{dt}$ are the rates of change of the prey and predator populations with respect to time. If the model time step in a year the equations represent the increase or decrease of the numbers of prey and predators over a year. This is a dynamic model, so the rates change as the model runs as a result of changes in both populations simultaneously. It is this aspect of the model that makes the dynamic interesting and hard to predict without running the model as a numerical simulation. You can't do the calculations in your head!

The R_1 parameter represents the intrinsic rate of growth of the prey population N in the absence of predation. If there were no predators then the prey population would increase exponentially. However it doesn't do this as prey are consumed by predators in the model.

The α parameter controls mortality due to hunting. The loss of prey is modelled by multiplying α by the number of prey and the number of predators. To understand this, think about the case when there is just one predator and 100 prey. If the predator can catch 10% of the prey (i.e. ten prey) then α would be 0.1.

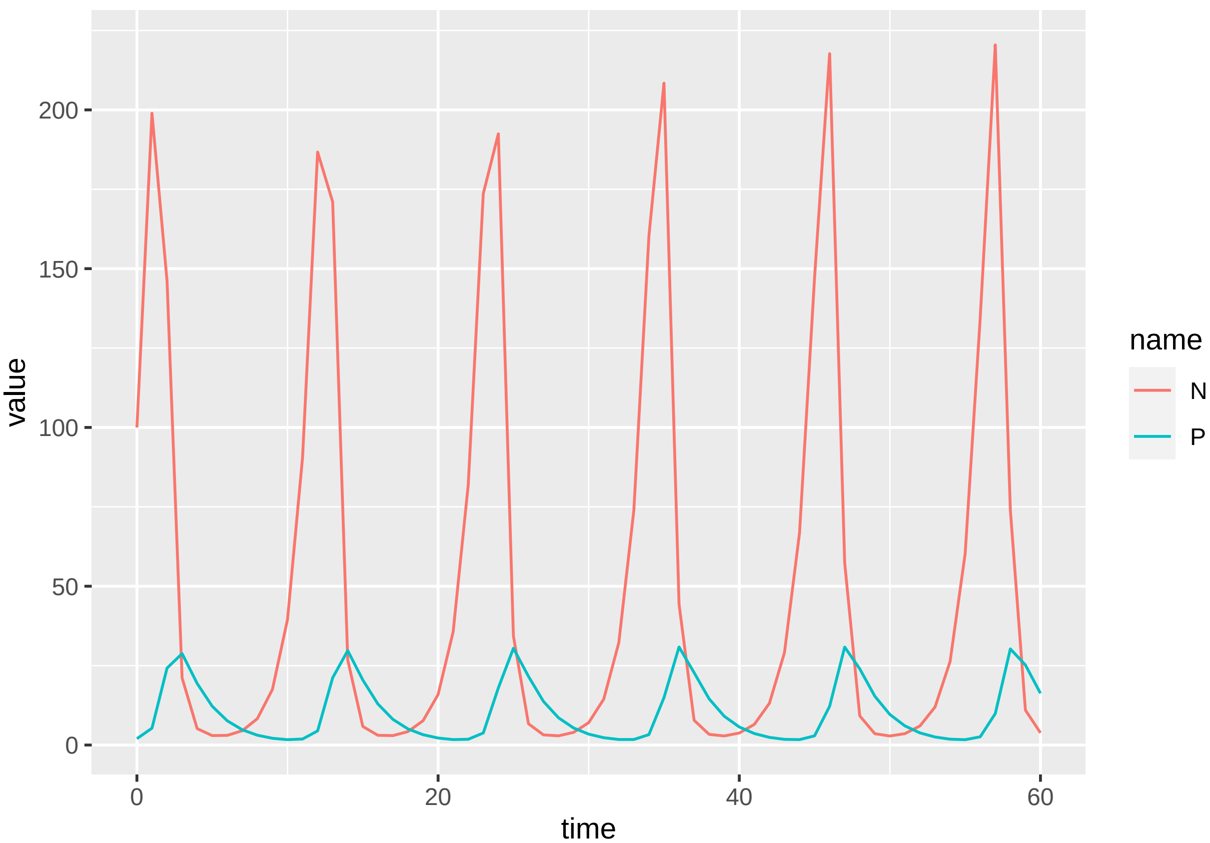
Once the prey are eaten they can be used to increase the predator population. The very simple model assumes that there is a conversion factor β that determines how many new prey are added to the population. So if ten prey are equivalent to ten new predators then β would equal 0.1

The predator population dies off through an exponential decay process modelled as γP. So if half the prey die each year γ would equal 0.5

Although the text book treatments of these equations can give the impression of sophistication and lead to beautiful mathematical treatments, in reality the model is very unstable. It is clearly oversimplified.

1. There is no limitation to the growth of the prey population apart from predation. In other words food availability and other similar limiting factors are not included.
2. Mortality of prey **only** takes place through predation.
3. Predators only consume one form of prey. So in the absence of all prey they would become extinct.
4. The mathematical model allows fractions of individuals. So prey populations can fall below one in the model.

In fact if the simple model is implemented in R using the easy to estimate parameters as stated above it behaves like this (code to run the model is embedded in this document).



These extreme cycles can be modified by changing the parameters, but the instability and lack of realism of the model means that it is not a suitable representation of a real system. It is a mathematical toy. The classic example of rather similar cycles found in nature is the Canadian snowshoe hare and Lynx data provided by trapping records. Although this has frequently been used to justify the model closer inspection of the data led to some suspicions. In fact the cycles didn't match the predictions at all as the peak in prey numbers occurred after the peak in predators. This led one author to suggest (rather tongue in cheek) that hares must eat Lynx! (Gilpin 1973).

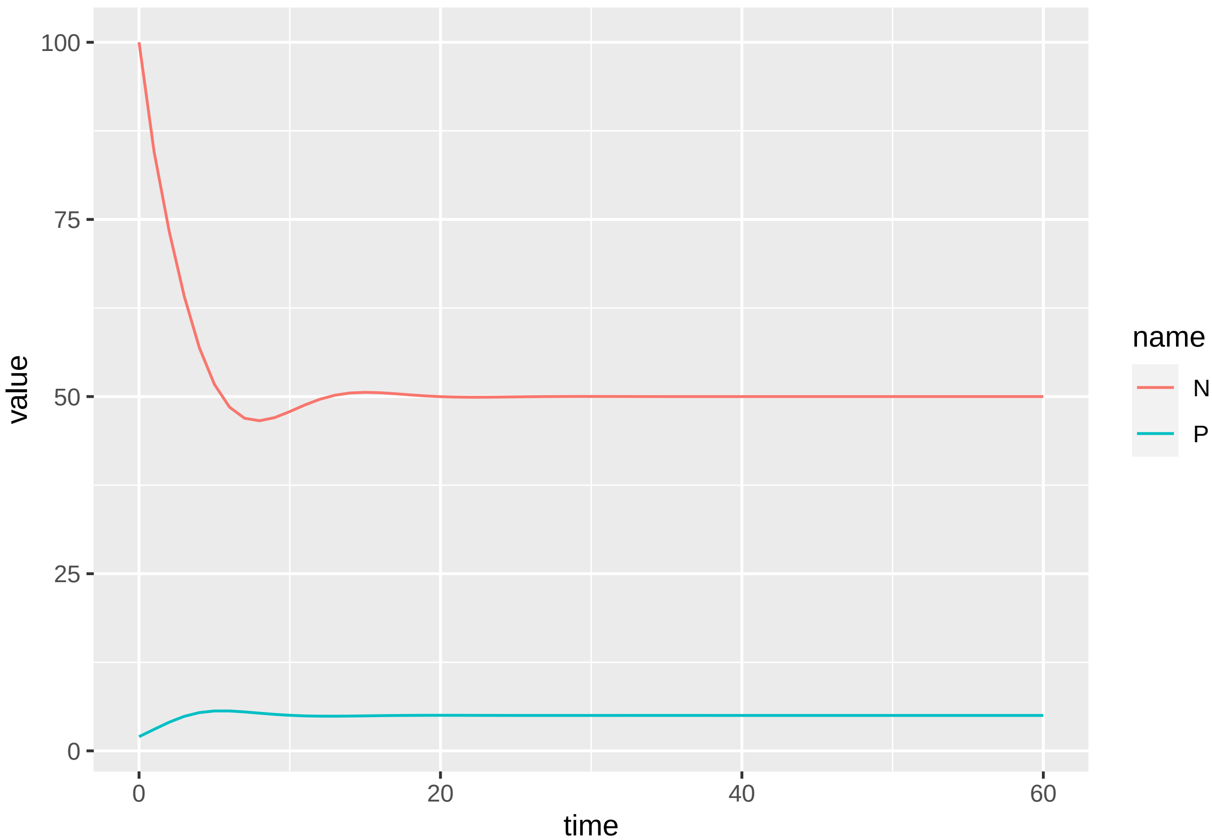
The Lotka Volterra model with prey carrying capacity

One element of the model that can be fixed quite easily is to assume that the prey population is controlled by some other environmental limitations. To do that we need to know the prey carrying capacity (K).

$\frac{dN}{dt} = r_1 N(1 - \frac{N}{K}) - \alpha NP$

$\frac{dP}{dt} = \alpha\beta NP - \gamma P$

Now if we run the model we get a much more boring result from the mathematical perspective.



The cyclical behaviour changes into one of long term equilibria between predator and prey. However this is now much more useful if we are thinking about how changing an ecosystem through introducing (or reintroducing) a predator may lead to an impact though the establishment of a new equilibrium prey population which is below the carrying capacity established by food availability. This may provide opportunities for the food to be exploited by another species and may stabilise boom and bust cycles in the prey.

Alternative prey

How can we take into consideration the possibility that the predator itself does not depend only on the prey species in order to survive? One simple way is to allow the predator to give birth to young without consuming the prey. Additional prey are a bonus. Consuming the prey species does add some predators, but the predator is not dependent on them. This can be quite easily added to the model.

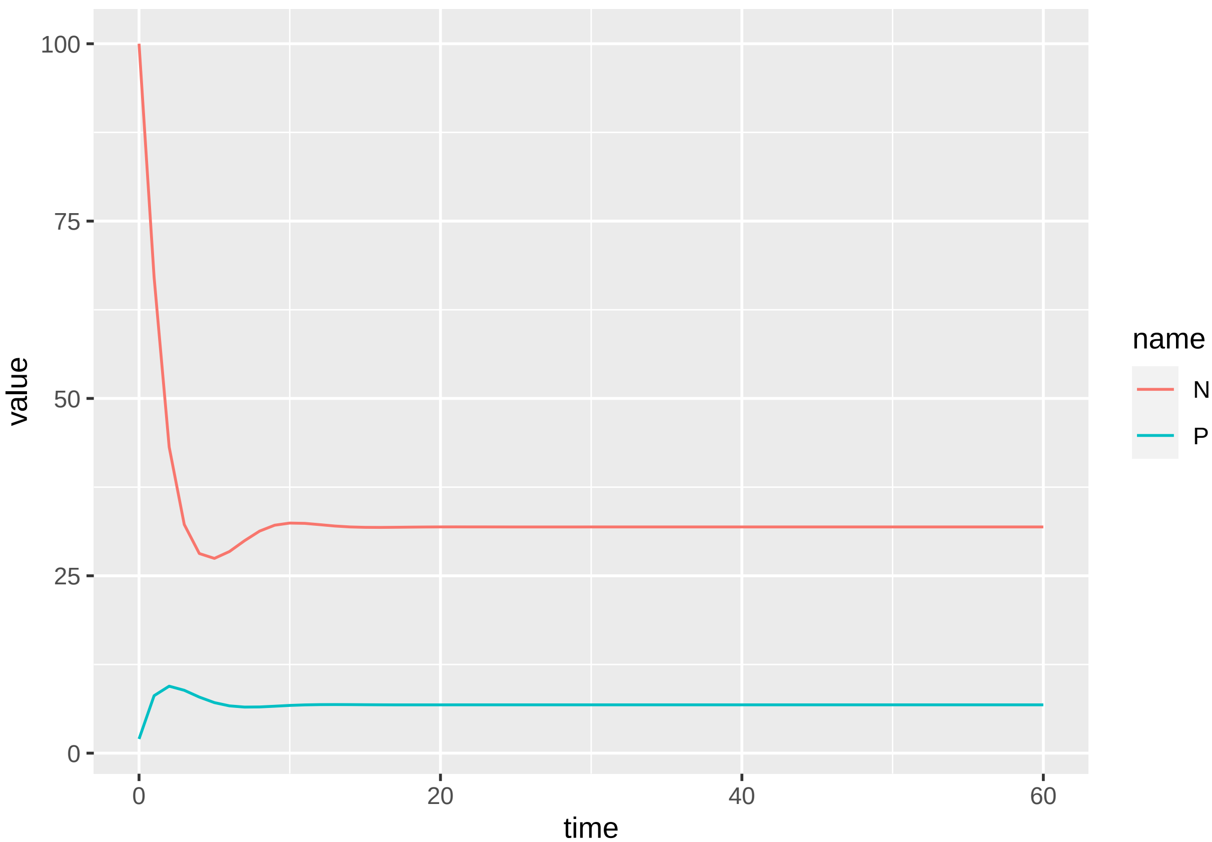
$\frac{dN}{dt} = r_1 N(1 - \frac{N}{K}) - \alpha NP$

$\frac{dP}{dt} = r_2 P + \alpha\beta NP - \gamma P$

A minor extension of this idea is to also add predator independent mortality of the prey species. This makes sense, as not all prey that die are consumed by the predator. This just requires adding another γ parameter to the equations.

$\frac{dN}{dt} = r_1 N(1 - \frac{N}{K}) - \alpha NP - \gamma_1 N$

$\frac{dP}{dt} = r_2 P + \alpha\beta NP - \gamma_2 P$



This is the essence of the model you will use for the project, expressed in mathematical terms. You can use the equations in the methods section in order to give more formality to the work. However if you don't feel comfortable with the dry formal mathematics, don't worry. The mathematical formulation remains very simple in essence. To develop a more intuitive model we will use the mathematics as a framework, and implement these ideas in a more expressive form using insight maker.

References

Berryman, Alan A. 1992. "The Orgins and Evolution of Predator-Prey Theory." *Ecology* 73 (5): 1530–35.

Ferguson, Neil M., Derek A. T. Cummings, Christophe Fraser, James C. Cajka, Philip C. Cooley, and Donald S. Burke. 2006. "Strategies for mitigating an influenza pandemic." *Nature* 442 (7101): 448–52. https://doi.org/10.1038/nature04795.

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